



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

NOVEMBER 2015

MARKS: 150

TIME: 3 hours

**This question paper consists of 14 pages, 1 information sheet
and a 25-page answer book.**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An INFORMATION SHEET with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The table below shows the total fat (in grams, rounded off to the nearest whole number) and energy (in kilojoules, rounded off to the nearest 100) of 10 items that are sold at a fast-food restaurant.

Fat (in grams)	9	14	25	8	12	31	28	14	29	20
Energy (in kilojoules)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

- 1.1 Represent the information above in a scatter plot on the grid provided in the ANSWER BOOK. (3)
- 1.2 The equation of the least squares regression line is $\hat{y} = 154,60 + 77,13x$.
- 1.2.1 An item at the restaurant contains 18 grams of fat. Calculate the number of kilojoules of energy that this item will provide. Give your answer rounded off to the nearest 100 kJ. (2)
- 1.2.2 Draw the least squares regression line on the scatter plot drawn for QUESTION 1.1. (2)
- 1.3 Identify an outlier in the data set. (1)
- 1.4 Calculate the value of the correlation coefficient. (2)
- 1.5 Comment on the strength of the relationship between the fat content and the number of kilojoules of energy. (1)
- [11]**

QUESTION 2

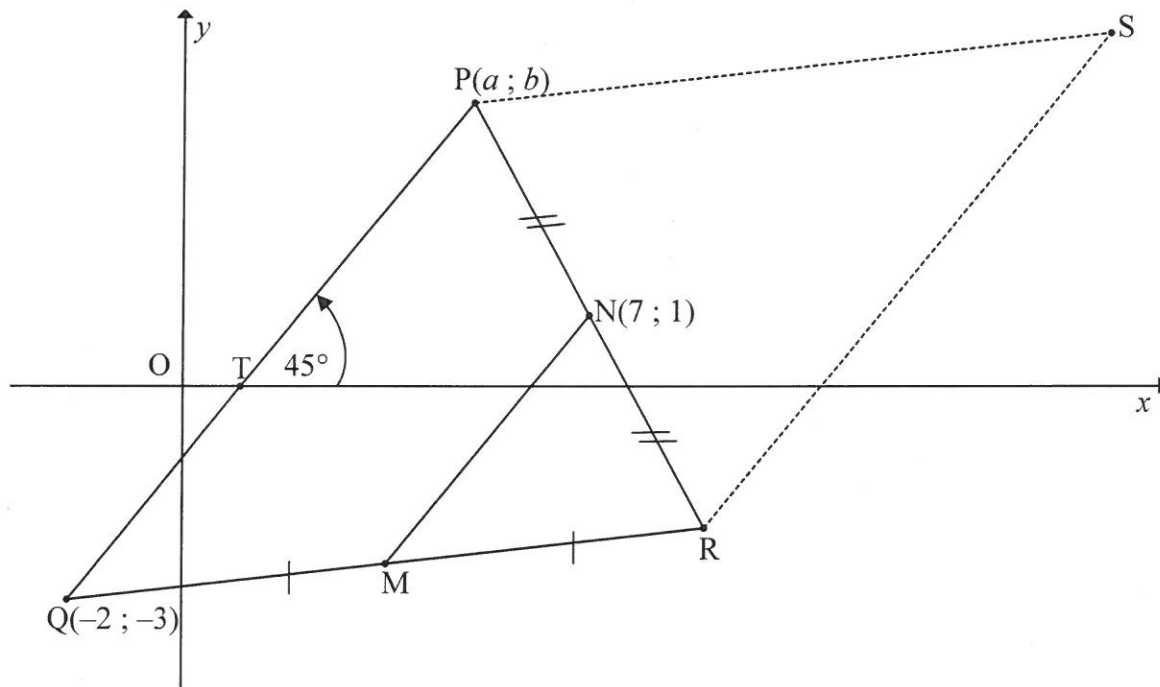
A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

- 2.1 Calculate the mean of the data. (2)
- 2.2 Determine the median of the data. (2)
- 2.3 Determine the standard deviation of the data. (2)
- 2.4 Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3)
- [9]

QUESTION 3

In the diagram below, the line joining $Q(-2; -3)$ and $P(a; b)$, a and $b > 0$, makes an angle of 45° with the positive x -axis. $QP = 7\sqrt{2}$ units. $N(7; 1)$ is the midpoint of PR and M is the midpoint of QR .



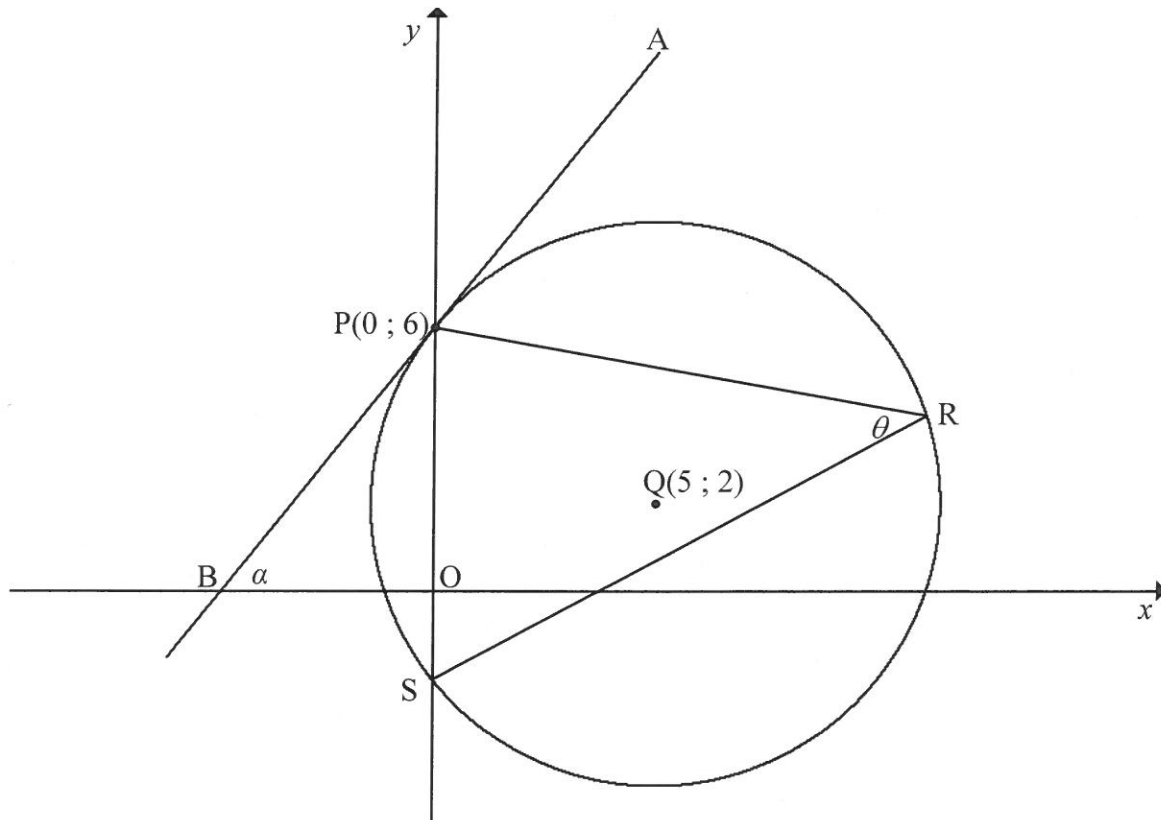
Determine:

- 3.1 The gradient of PQ (2)
- 3.2 The equation of MN in the form $y = mx + c$ and give reasons (4)
- 3.3 The length of MN (2)
- 3.4 The length of RS (1)
- 3.5 The coordinates of S such that $PQRS$, in this order, is a parallelogram (3)
- 3.6 The coordinates of P (6)

[18]

QUESTION 4

In the diagram below, $Q(5 ; 2)$ is the centre of a circle that intersects the y -axis at $P(0 ; 6)$ and S . The tangent APB at P intersects the x -axis at B and makes the angle α with the positive x -axis. R is a point on the circle and $\widehat{PRS} = \theta$.



- 4.1 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)
- 4.2 Calculate the coordinates of S . (3)
- 4.3 Determine the equation of the tangent APB in the form $y = mx + c$. (4)
- 4.4 Calculate the size of α . (2)
- 4.5 Calculate, with reasons, the size of θ . (4)
- 4.6 Calculate the area of $\triangle PQS$. (4)

[20]

QUESTION 5

5.1 Given that $\sin 23^\circ = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k , WITHOUT using a calculator:

5.1.1 $\sin 203^\circ$ (2)

5.1.2 $\cos 23^\circ$ (3)

5.1.3 $\tan(-23^\circ)$ (2)

5.2 Simplify the following expression to a single trigonometric function:

$$\frac{4 \cos(-x) \cdot \cos(90^\circ + x)}{\sin(30^\circ - x) \cdot \cos x + \cos(30^\circ - x) \cdot \sin x} \quad (6)$$

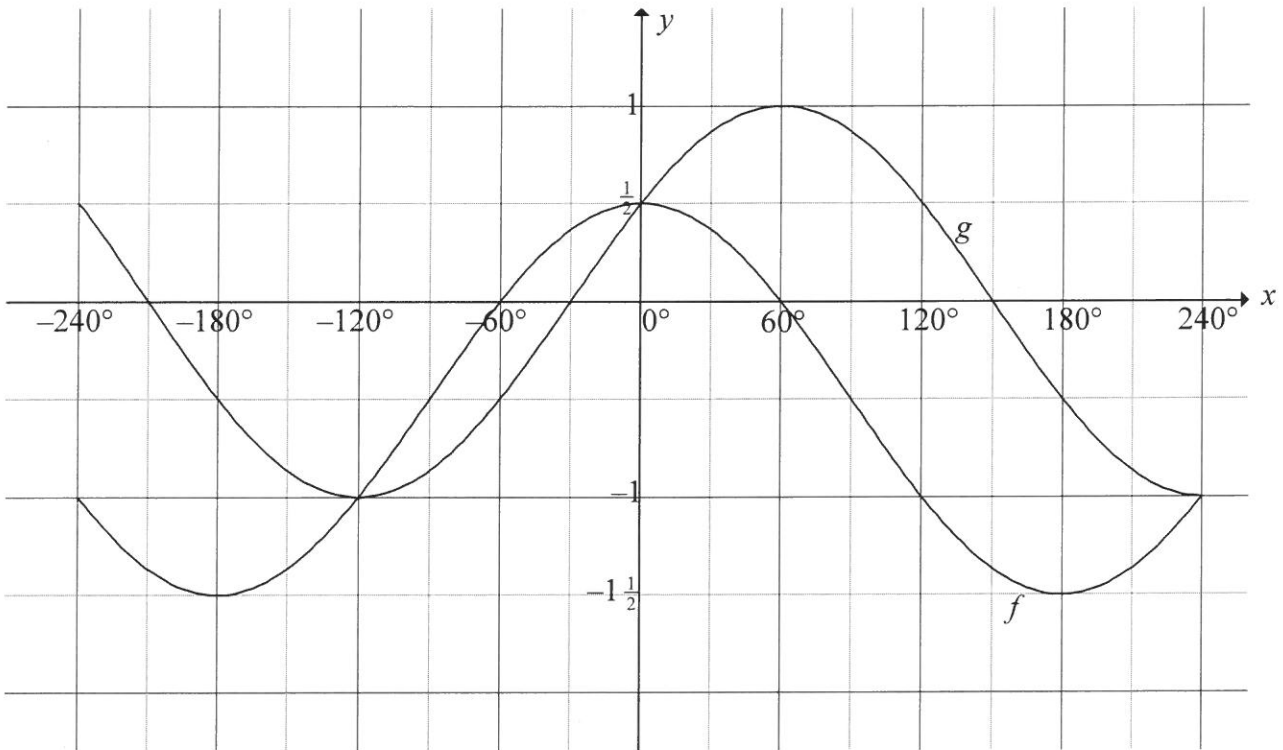
5.3 Determine the general solution of $\cos 2x - 7 \cos x - 3 = 0$. (6)

5.4 Given that $\sin \theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)

[24]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240^\circ \leq x \leq 240^\circ$. The graphs intersect at $(0^\circ; \frac{1}{2})$, $(-120^\circ; -1)$ and $(240^\circ; -1)$.



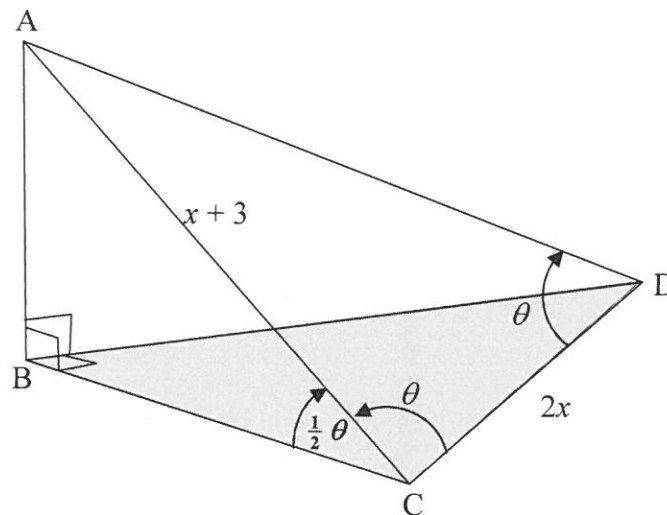
- 6.1 Determine the values of p and q . (4)
 - 6.2 Determine the values of x in the interval $-240^\circ \leq x \leq 240^\circ$ for which $f(x) > g(x)$. (2)
 - 6.3 Describe a transformation that the graph of g has to undergo to form the graph of h , where $h(x) = -\cos x$. (2)
- [8]**

QUESTION 7

A corner of a rectangular block of wood is cut off and shown in the diagram below.

The inclined plane, that is, $\triangle ACD$, is an isosceles triangle having $\hat{ADC} = \hat{ACD} = \theta$.

Also $\hat{ACB} = \frac{1}{2}\theta$, $AC = x + 3$ and $CD = 2x$.

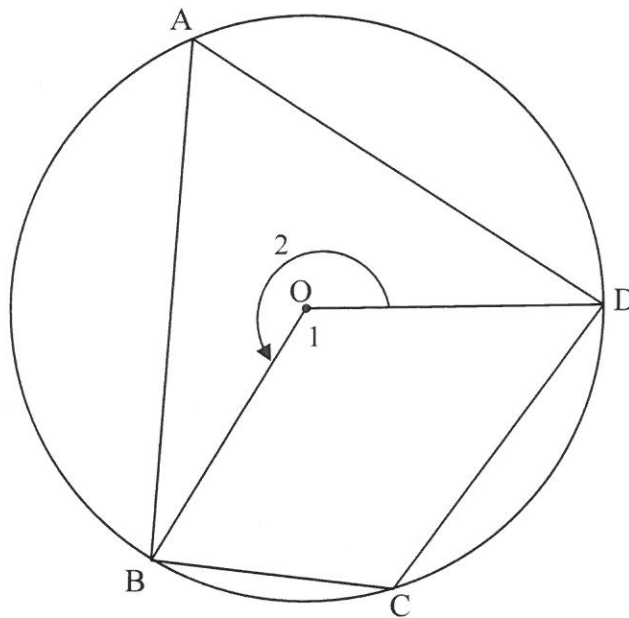


- 7.1 Determine an expression for \hat{CAD} in terms of θ . (1)
- 7.2 Prove that $\cos \theta = \frac{x}{x+3}$. (4)
- 7.3 If it is given that $x = 2$, calculate AB , the height of the piece of wood. (5)
- [10]**

Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

8.1 In the diagram below, cyclic quadrilateral $ABCD$ is drawn in the circle with centre O .

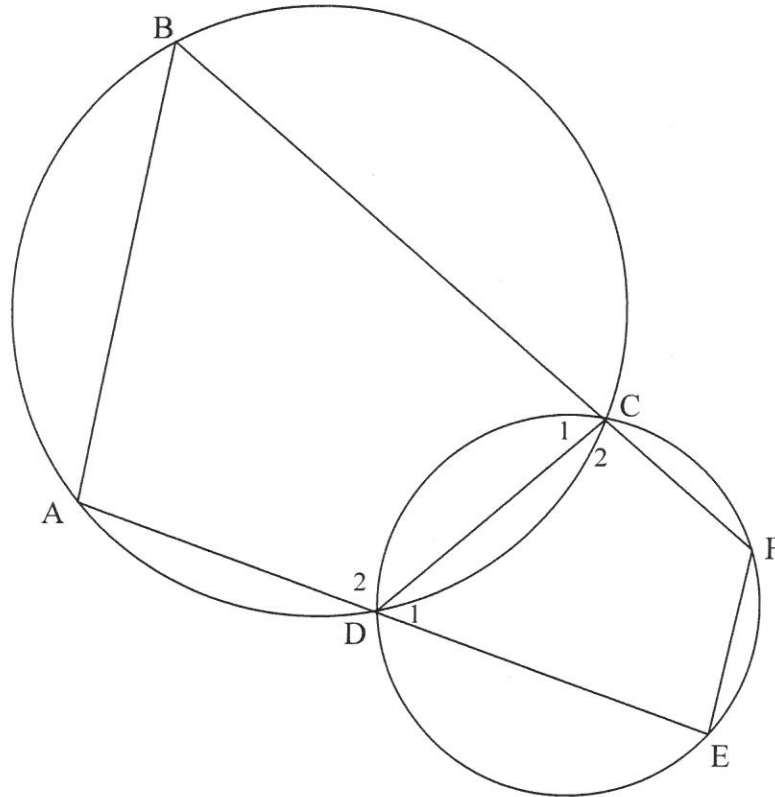


8.1.1 Complete the following statement:

The angle subtended by a chord at the centre of a circle is ... the angle subtended by the same chord at the circumference of the circle. (1)

8.1.2 Use QUESTION 8.1.1 to prove that $\hat{A} + \hat{C} = 180^\circ$. (3)

8.2 In the diagram below, CD is a common chord of the two circles. Straight lines ADE and BCF are drawn. Chords AB and EF are drawn.

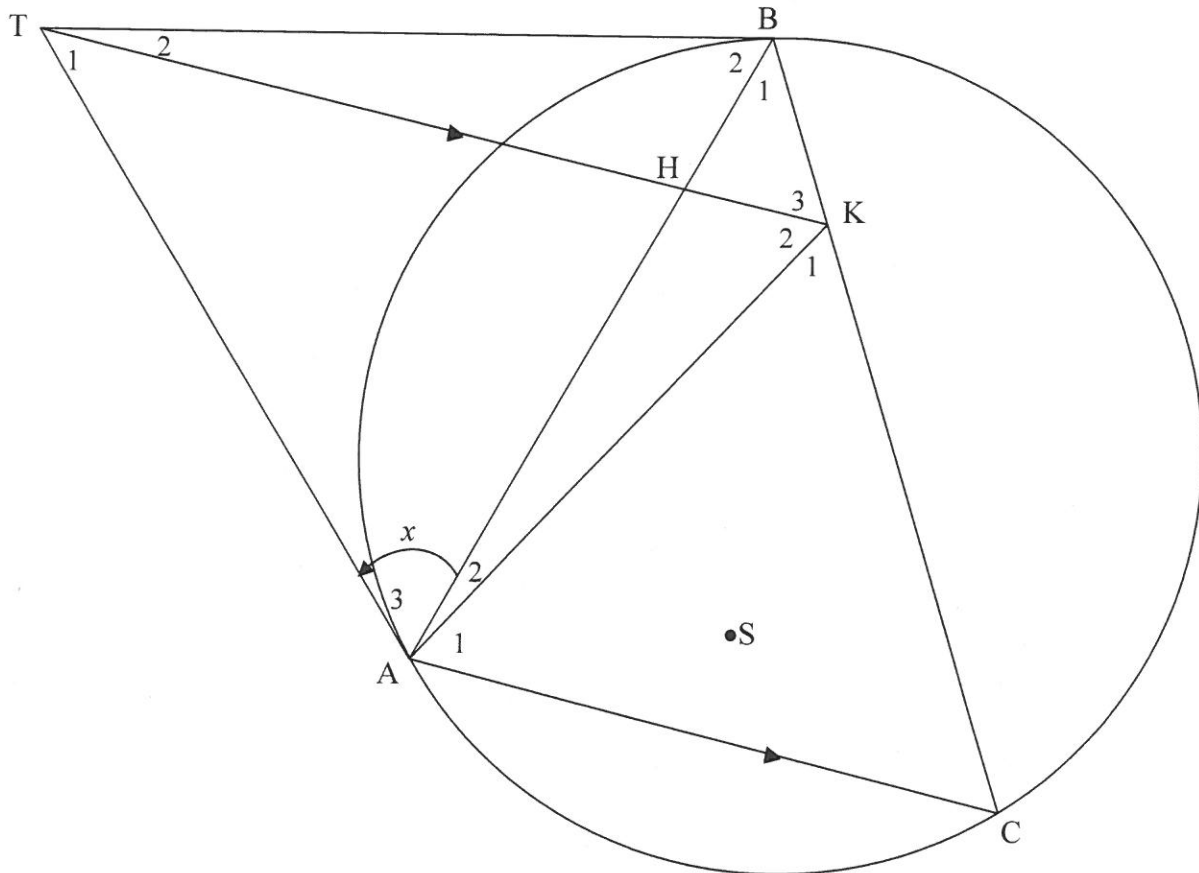


Prove that $EF \parallel AB$.

(5)
[9]

QUESTION 9

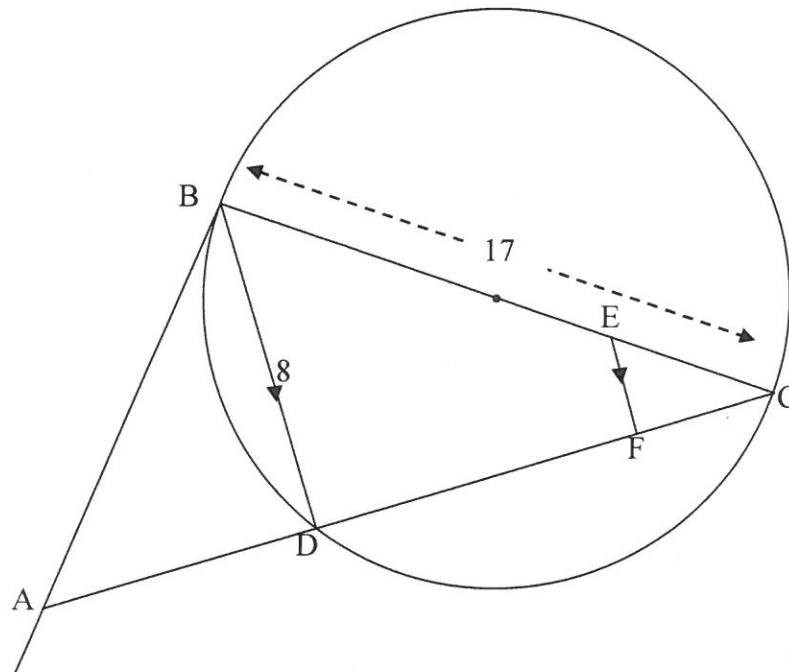
In the diagram below, $\triangle ABC$ is drawn in the circle. TA and TB are tangents to the circle. The straight line THK is parallel to AC with H on BA and K on BC . AK is drawn. Let $\hat{A}_3 = x$.



- 9.1 Prove that $\hat{K}_3 = x$. (4)
 - 9.2 Prove that $AKBT$ is a cyclic quadrilateral. (2)
 - 9.3 Prove that TK bisects \hat{AKB} . (4)
 - 9.4 Prove that TA is a tangent to the circle passing through the points A , K and H . (2)
 - 9.5 S is a point in the circle such that the points A , S , K and B are concyclic. Explain why A , S , B and T are also concyclic. (2)
- [14]**

QUESTION 10

In the diagram below, $BC = 17$ units, where BC is a diameter of the circle. The length of chord BD is 8 units. The tangent at B meets CD produced at A .



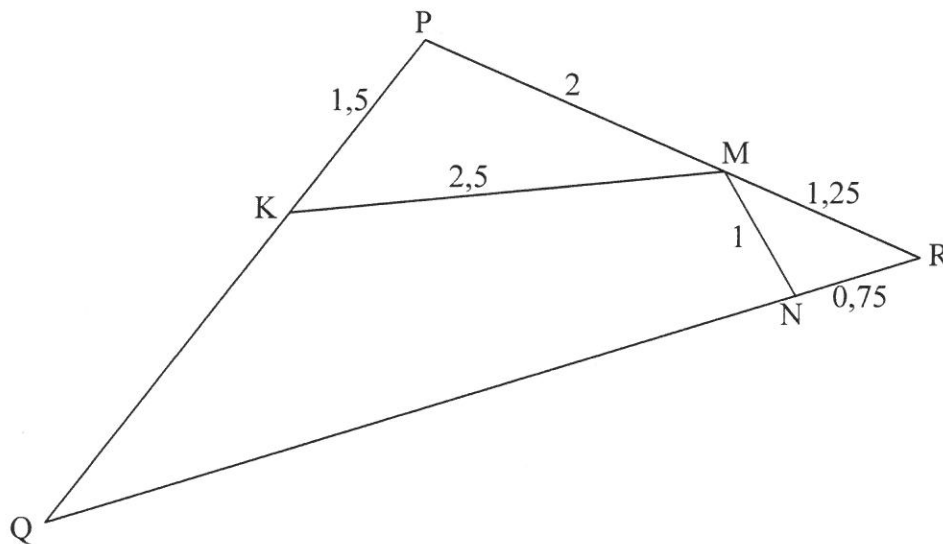
- 10.1 Calculate, with reasons, the length of DC . (3)
 - 10.2 E is a point on BC such that $BE : EC = 3 : 1$. EF is parallel to BD with F on DC .
 - 10.2.1 Calculate, with reasons, the length of CF . (3)
 - 10.2.2 Prove that $\triangle BAC \sim \triangle FEC$. (5)
 - 10.2.3 Calculate the length of AC . (4)
 - 10.2.4 Write down, giving reasons, the radius of the circle passing through points A , B and C . (2)
- [17]**

QUESTION 11

11.1 Complete the following statement:

If the sides of two triangles are in the same proportion, then the triangles are ... (1)

11.2 In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of $\triangle PQR$. $KP = 1,5$; $PM = 2$; $KM = 2,5$; $MN = 1$; $MR = 1,25$ and $NR = 0,75$.



11.2.1 Prove that $\triangle KPM \sim \triangle RNM$. (3)

11.2.2 Determine the length of NQ. (6)
[10]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$